

Nonuniversality of indirect CP asymmetries in $D \rightarrow \pi\pi, KK$ decays

Amol Dighe ^{*1}, Diptimoy Ghosh ^{†2}, and Bhavik Kodrani^{‡1}

¹Tata Institute of Fundamental Research, Mumbai 400005, India

²INFN, Sezione di Roma, Piazzale A. Moro 2, I-00185, Roma, Italy

Abstract

We point out that, if the direct CP asymmetries in the $D \rightarrow \pi^+\pi^-$ and $D \rightarrow K^+K^-$ decays are unequal, the indirect CP asymmetries as measured in these modes are necessarily unequal. This nonuniversality of indirect CP asymmetries can be significant with the right amount of new physics contributions, a scenario that may be fine-tuned, but is still viable. A model-independent fit to the current data allows different indirect CP asymmetries in the above two decays. This could even be contributing to the apparent tension between the difference CP asymmetries ΔA_{CP} measured through the pion-tagged and muon-tagged data samples at the LHCb. This also implies that the measurements of A_Γ and y_{CP} in the $\pi^+\pi^-$ and K^+K^- decay modes can be different, and averaging over these two modes should be avoided. In any case, the complete analysis of CP violation measurements in the D meson sector needs to take into account the possibility of different indirect CP asymmetries in the $\pi^+\pi^-$ and K^+K^- channels.

^{*}amol@theory.tifr.res.in

[†]diptimoy.ghosh@roma1.infn.it

[‡]bhavik@theory.tifr.res.in

1 Introduction

The study of charge-parity (CP) violation in decays of K and B mesons have yielded path-breaking results over the past half a century. Through these measurements, the Kobayashi-Maskawa paradigm of CP violation has been tested from many directions, and has emerged vindicated so far. These tests act as indirect probes of new physics beyond the Standard Model, and so far have not yielded any conclusive evidence for a deviation from the Standard Model (SM) predictions. A positive identification of deviations from the SM is often limited by the uncertainties in the SM predictions themselves, especially in the processes that involve decays of mesons, due to hadronic uncertainties.

The measurements of $D - \bar{D}$ mixing and the CP violation in D decays have started yielding interesting results only in the past decade. One of the reasons for the D decays to have come to the forefront so late is that the mixing as well as CP violation in the D sector is expected to be small. In the SM, the contribution to the $D - \bar{D}$ mixing box diagram is suppressed — for intermediate u and c quarks, due to their small masses, and for an intermediate b quark, due to the small Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. This leads to very small values for both the dispersive as well as absorptive parts of the $D - \bar{D}$ mixing amplitude. Indeed, the measurements give $x \equiv \Delta m/\Gamma$ and $y \equiv \Delta\Gamma/(2\Gamma)$ to be less than $\mathcal{O}(1\%)$ [1, 2, 3]. Moreover, since the phases of the relevant CKM matrix elements are very small, the CP violation is also expected to be not more than $\mathcal{O}(0.1\%)$ [4].

The SM calculations of the mixing and CP asymmetries in the neutral D meson system are difficult due to the mass of the charm quark — it is not light enough to enable the use of the chiral perturbation theory, and not heavy enough to guarantee convergence of the $(1/m_c)$ expansion in the heavy quark effective theory. Moreover, the nonperturbative long-distance contributions to the mixing as well as decay amplitudes may be dominant — since the strong coupling is not very small at the scale of the D meson mass, and the short-distance contributions do not have the benefit of an intermediate t quark as in the case of B mesons. Therefore, D decays are not a good place for precision measurements of the SM parameters. However, they can still be used as probes of new physics (NP), if the NP effects can be large compared to the SM ones [4, 5]. Modes like $D/\bar{D} \rightarrow \pi^+\pi^-$ and $D/\bar{D} \rightarrow K^+K^-$ can be sensitive to the presence of such NP [6, 7].

While the measurements of the mixing parameters x and y are consistent with the SM estimates of $x, y \sim \mathcal{O}(1\%)$ [8, 9], recent measurements of CP-violating quantities have given us a reason to consider the presence of NP contributions. The CP violation in $D \rightarrow \pi^+\pi^-$ and $D \rightarrow K^+K^-$ decays was constrained by E791 [10], FOCUS [11], CLEO [12], and the B factories [13, 14]. Recently CDF [15, 16] and LHCb [17, 18, 19] presented the measurements for the “difference CP asymmetry” ΔA_{CP} , the difference between the CP asymmetries in the above two decay modes, that was expected to cancel out some of the systematic uncertainties. These results, obtained using the pion-tagged samples, indicated a value of $\mathcal{O}(0.5\%)$ for ΔA_{CP} . These measurements disfavored a vanishing CP asymmetry, and were also away from the SM prediction (leading order in $1/m_c$ [21]) of $\mathcal{O}(0.05\% - 0.1\%)$ by more than $\sim 2\sigma$. Moreover, the latest LHCb results with the pion-tagged sample [18] and the muon-tagged sample [19] have central values with opposite signs, and differ by $\sim 2.2\sigma$ from each other. The average of these two LHCb measurements has also

been recently reported [20], which is consistent with vanishing ΔA_{CP} . (However as we shall point out in this paper, such an average need not be the right observable to look for.)

Several attempts have been made to check if the observed large CP asymmetries can be accommodated within the SM. It has been claimed that QCD penguin operators, with large strong phases, may give rise to a significant enhancement [21, 22, 23, 24]. The breaking of $SU(3)$ [25], or U -spin [26, 27] symmetries, or of the naive $1/N_c$ counting [28] may also be a reason for the observed large ΔA_{CP} . However the jury is still out on whether these contributions can account for the data without the need to go beyond the SM.

Specific NP models that can enhance the CP asymmetries have also been extensively studied. These include the fourth quark generation [29, 27], supersymmetric gluino-squark loops [6], littlest Higgs model with T-parity [30, 31], flavor violation in the up sector [32, 33], models with a color-sextet diquark [34], models giving rise to the t -channel exchange of a weak doublet with a special flavor structure [35], the nonmanifest left-right symmetric model [36], or models with warped extra dimensions [37]. A survey of the effect of NP models that may contribute to the difference CP asymmetry has been performed in Ref. [38]. It points out that the CP violation may be generated at the tree level with models that involve flavor-changing couplings of Z, Z' bosons, new charged gauge bosons, flavor-changing heavy gluon, scalar octets, a scalar diquark, or a two-Higgs doublet with minimal flavor violation. Models with GIM-unsuppressed fermion and scalar loops, or those with chirally enhanced magnetic penguin operators, can also contribute to the CP asymmetry at the loop level. It has been observed [39] that NP models in which the primary source of flavor violation is linked to the breaking of chiral symmetry are natural candidates to explain the CP asymmetries, via enhanced chromomagnetic operators. Many of these models also affect the measurements of other D decay channels, as well as the $D - \bar{D}$ mixing and ϵ'/ϵ in the K sector, and hence the masses of new particles and couplings in most of these models are severely constrained [40].

Identifying whether the enhancement in the CP violation in $D \rightarrow \pi^+\pi^-, K^+K^-$ is from the SM or NP is not straightforward; however, some information may be obtained from related decay modes. It was pointed out in Ref. [41] that, since the enhancement due to nonperturbative physics should only affect exclusive modes, an enhancement in the inclusive modes will point definitively to NP. One could also look at modes related to $\pi^+\pi^-, K^+K^-$ by isospin symmetry, since this symmetry is not expected to be broken significantly. Such a comparison could distinguish between a large penguin amplitude and an enhanced chromomagnetic dipole operator, for example Ref. [42].

The aim of this paper is not to check whether the SM or any specific NP model explains the data. Rather, we choose to take the data at face value, and learn in a model-independent way what they tell us about the CP violation in the $D - \bar{D}$ mixing and decay. To this end, we perform a fit to the data with four complex parameters, $M_{12}, \Gamma_{12}, R_\pi$ and R_K . Here, the mixing parameters M_{12} and Γ_{12} are the complex-valued dispersive and absorptive components, respectively, of the effective $D - \bar{D}$ mixing Hamiltonian. The other two parameters,

$$R_\pi \equiv \frac{A(\bar{D} \rightarrow \pi^+\pi^-)}{A(D \rightarrow \pi^+\pi^-)} \quad \text{and} \quad R_K \equiv \frac{A(\bar{D} \rightarrow K^+K^-)}{A(D \rightarrow K^+K^-)},$$

are the ratios of decay amplitudes of a pure \bar{D} and D to the CP eigenstates.

The data on $D - \bar{D}$ mixing and CP asymmetries in the $\pi^+\pi^-$ and K^+K^- channels form the main input for the fit. The ingredients for the fit also include the asymmetries $A_\Gamma(\pi)$, $A_\Gamma(K)$, $y_{\text{CP}}(\pi)$, and $y_{\text{CP}}(K)$, constructed from the ratios of effective lifetimes measured in the CP-eigenstate modes $D/\bar{D} \rightarrow \pi^+\pi^-$, K^+K^- , and the (almost-)flavor-specific modes $D \rightarrow K^+\pi^-$, $\bar{D} \rightarrow K^-\pi^+$ [43], which have been reported by FOCUS [44], CLEO [12], Belle [45], Babar [46, 47], and LHCb [48]. While the measurements of $A_\Gamma(\pi)$, $A_\Gamma(K)$ and $y_{\text{CP}}(\pi)$ are consistent with zero to within 2σ , the asymmetry $y_{\text{CP}}(K)$ has been found to be nonzero to more than 4σ [45, 46, 47]. More recently, Belle [49] and Babar [50] have reported values of A_Γ and y_{CP} averaged over the $\pi^+\pi^-$ and K^+K^- samples, where the $y_{\text{CP}}(\text{avg})$ has been found to be nonzero to more than $\sim 4\sigma$ in each experiment. These asymmetries may be represented in terms of the combinations of the same parameters considered above, hence the information content in these measurements is also relevant in determining the favored parameter values, and in fact, is commonly used [3].

The classification of CP violation in neutral meson systems is normally described in two languages. One may talk in terms of CP violation in only mixing (deviation of $|q/p|$ from unity), in only decay (deviation of $|R_f|$ from unity, where $R_f \equiv \bar{A}_f/A_f$), and in the interference of mixing and decay (imaginary part of $\lambda_f \equiv (q/p)R_f$). This is the standard notation used in the discussion of B decays. On the other hand, one may use the language of direct vs. indirect CP violation, which has its origins in the analyses of K decays. While we personally prefer the former formulation due to its clarity in distinguishing the source of the CP violation, the latter one has been used in most of the literature on the CP asymmetries in D decays that is the focus of this paper. Indeed, the recent experimental data [15, 16, 17, 18, 19] have been interpreted in terms of the direct and indirect CP asymmetries ($A_{\text{CP}}^{\text{dir}}$ and $A_{\text{CP}}^{\text{indir}}$, respectively) in the $\pi\pi$ and KK decays. We therefore shall refer to both the notations, at the risk of some repetition in presenting our results and interpretations.

The interpretation of ΔA_{CP} in terms of its direct and indirect components often [16, 17, 18, 19] takes $A_{\text{CP}}^{\text{indir}}(\pi) = A_{\text{CP}}^{\text{indir}}(K)$. We point out that if this condition were strictly valid, it would also imply $A_{\text{CP}}^{\text{dir}}(\pi) = A_{\text{CP}}^{\text{dir}}(K)$, independent of the origin of the CP asymmetry. This is clearly not the case, even in the limit of flavor $SU(3)$ where these two quantities have the same magnitudes but opposite signs. Therefore, the assumption of exactly equal $A_{\text{CP}}^{\text{indir}}$ in the $\pi^+\pi^-$ and K^+K^- channels is, strictly speaking, not accurate. In practice, with certain “natural” expectations about the amplitudes and phases of NP contributions, the nonuniversality of $A_{\text{CP}}^{\text{indir}}$ may turn out to be so small that it may be neglected [6, 7], since the difference $A_{\text{CP}}^{\text{dir}}(\pi) - A_{\text{CP}}^{\text{dir}}(K)$ is less than $\mathcal{O}(0.01)$ and is expected to contribute to the nonuniversality $A_{\text{CP}}^{\text{indir}}(\pi) = A_{\text{CP}}^{\text{indir}}(K)$ only to the second order. However, while searching for physics beyond the SM, the analysis of data should be performed without prejudice to theoretical expectations, and alternative scenarios, however unlikely they may seem, should be considered. We therefore reanalyze the current data without the approximation $A_{\text{CP}}^{\text{indir}}(\pi) = A_{\text{CP}}^{\text{indir}}(K)$. Our fit, in fact, shows that the preferred parameter space allows significantly different values for $A_{\text{CP}}^{\text{indir}}$ in $\pi^+\pi^-$ and K^+K^- decays. Such a difference could also contribute to the seemingly different ΔA_{CP} values measured through the pion-tagged and muon-tagged data samples at the LHCb [18, 19]. That this nonuniversality also leads to the nonuniversality of A_Γ and y_{CP} has been indirectly alluded to in Ref. [51].

Our paper is organized as follows. In Sec. 2, we present the analytical expressions for the time-dependent CP asymmetries and their direct and indirect components, $A_{\text{CP}}^{\text{dir}}$ and $A_{\text{CP}}^{\text{indir}}$, inferred from the data. We also relate A_{Γ} and y_{CP} , the quantities obtained from the measurements of effective D decay rates in different channels, to the relevant CP-violating quantities. In Sec. 3, we perform a χ^2 fit to the data and obtain the favored values for the parameters of interest. Section 4 is devoted to the feasibility and implications of a significant nonuniversality of $A_{\text{CP}}^{\text{indir}}$. Section 5 summarizes our results and recommends taking the possible nonuniversality in $A_{\text{CP}}^{\text{indir}}$ into account for future analyses of neutral D decay data.

2 $D - \bar{D}$ mixing and decay: formalism

We follow the analysis of $D - \bar{D}$ mixing and decay as in Refs. [4, 51]. Since the notations vary from analysis to analysis, we repeat the relevant steps to clarify our notation. In the (D, \bar{D}) flavor basis, the effective Hamiltonian $H = M - i\Gamma/2$ is not diagonal. The off-diagonal elements of the dispersive and absorptive components, i.e. M_{12} and Γ_{12} , are responsible for the $D - \bar{D}$ mixing. The mass eigenstates are given by

$$|D_L\rangle = p|D\rangle + q|\bar{D}\rangle, \quad |D_H\rangle = p|D\rangle - q|\bar{D}\rangle, \quad (1)$$

where

$$|q|^2 + |p|^2 = 1, \quad \left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}. \quad (2)$$

The deviation of $|q/p|$ from unity corresponds to CP violation in mixing. Note that as opposed to the mixing in the neutral B systems ($B - \bar{B}, B_s - \bar{B}_s$) where $|\Gamma_{12}| \ll |M_{12}|$, here we have the possibility of Γ_{12} being of the same order as M_{12} or even a few times larger [43]. If in addition, M_{12} and Γ_{12} have significantly different phases, then $|q/p|$ can differ substantially from unity, and the effects of this CP violation in mixing need to be taken care of in the analysis.

With the mass difference and the decay width difference of the interaction eigenstates $D_{H,L}$ defined as

$$\Delta m = m_H - m_L, \quad \Delta\Gamma = \Gamma_H - \Gamma_L, \quad (3)$$

the time evolutions of the mass eigenstates are

$$|D_{H,L}(t)\rangle = e^{-i(m_{H,L} - \frac{i}{2}\Gamma_{H,L})t} |D_{H,L}\rangle. \quad (4)$$

Using Eq. (1) and Eq. (4), the time evolution of an initial D or \bar{D} state becomes

$$|D(t)\rangle = g_+(t)|D\rangle - \frac{q}{p}g_-(t)|\bar{D}\rangle, \quad (5)$$

$$|\bar{D}(t)\rangle = g_+(t)|\bar{D}\rangle - \frac{p}{q}g_-(t)|D\rangle, \quad (6)$$

where the coefficients $g_{\pm}(t)$ are

$$g_{\pm}(t) = \frac{1}{2} \left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t} \right). \quad (7)$$

We use the standard convention

$$A_f = \langle f|H|D \rangle, \quad \bar{A}_f = \langle f|H|\bar{D} \rangle \quad (8)$$

to denote the amplitudes for the decay of D and \bar{D} mesons to a final state f . The time-dependent decay rate of an initial D meson to the final state f can then be written as

$$\frac{d\Gamma(D(t) \rightarrow f)}{dt} = N_f |\langle f|H|D(t) \rangle|^2, \quad (9)$$

where N_f is the time-independent normalization factor. Using Eqs. (5), (6), and (7), we get

$$\begin{aligned} \frac{d\Gamma(D^0(t) \rightarrow f)}{dt} &= \frac{N_f}{2} e^{-\Gamma t} |A_f|^2 \times \\ &\quad \left[(1 + |\lambda_f|^2) \cosh(y\Gamma t) + (1 - |\lambda_f|^2) \cos(x\Gamma t) \right. \\ &\quad \left. + 2 \operatorname{Re}(\lambda_f) \sinh(y\Gamma t) - 2 \operatorname{Im}(\lambda_f) \sin(x\Gamma t) \right], \end{aligned} \quad (10)$$

where $\Gamma = (\Gamma_H + \Gamma_L)/2$, and $\lambda_f = (q/p)(\bar{A}_f/A_f)$. Similarly,

$$\begin{aligned} \frac{d\Gamma(\bar{D}(t) \rightarrow f)}{dt} &= \frac{N_f}{2} e^{-\Gamma t} \left| \frac{p}{q} A_f \right|^2 \times \\ &\quad \left[(1 + |\lambda_f|^2) \cosh(y\Gamma t) - (1 - |\lambda_f|^2) \cos(x\Gamma t) \right. \\ &\quad \left. + 2 \operatorname{Re}(\lambda_f) \sinh(y\Gamma t) + 2 \operatorname{Im}(\lambda_f) \sin(x\Gamma t) \right]. \end{aligned} \quad (11)$$

The expressions above are applicable for both the final states, $f = \pi^+\pi^-$ and $f = K^+K^-$, that are the focus of this paper.

2.1 Direct and indirect CP asymmetries

The time-dependent CP asymmetry for the decay process $D \rightarrow f$ is

$$A_{CP}(t) = \frac{\frac{d\Gamma(D(t) \rightarrow f)}{dt} - \frac{d\Gamma(\bar{D}(t) \rightarrow f)}{dt}}{\frac{d\Gamma(D(t) \rightarrow f)}{dt} + \frac{d\Gamma(\bar{D}(t) \rightarrow f)}{dt}}. \quad (12)$$

From Eqs. (10) and (11), we get

$$A_{CP}(t) = \frac{\left(\left| \frac{q}{p} \right|^2 - 1 \right) \Omega_+ + \left(\left| \frac{q}{p} \right|^2 + 1 \right) \Omega_-}{\left(\left| \frac{q}{p} \right|^2 + 1 \right) \Omega_+ + \left(\left| \frac{q}{p} \right|^2 - 1 \right) \Omega_-}, \quad (13)$$

where

$$\begin{aligned} \Omega_+ &\equiv (1 + |\lambda_f|^2) \cosh(y\Gamma t) + 2 \operatorname{Re}(\lambda_f) \sinh(y\Gamma t), \\ \Omega_- &\equiv (1 - |\lambda_f|^2) \cos(x\Gamma t) - 2 \operatorname{Im}(\lambda_f) \sin(x\Gamma t). \end{aligned} \quad (14)$$

Since $x, y \lesssim \mathcal{O}(1\%)$ [3] and $\Gamma t \sim \mathcal{O}(1)$ or less, the above exact expression may be simplified by expanding in the small parameters x and y , and keeping the leading terms. For convenience, we also use the notation

$$|q/p|^2 = 1 + \zeta. \quad (15)$$

Given the current 95% bounds $0.44 < |q/p| < 1.07$ [3], we cannot take ζ to be a small quantity. The expansion in small parameters x and y to linear order allows us to write Eq. (13) in the form¹

$$A_{\text{CP}}(t) = A_{\text{CP}}^{\text{dir}} + \frac{t}{\tau_D} A_{\text{CP}}^{\text{indir}}, \quad (16)$$

where τ_D is the lifetime of the D meson. Here $A_{\text{CP}}^{\text{dir}}$ and $A_{\text{CP}}^{\text{indir}}$ are given as

$$A_{\text{CP}}^{\text{dir}} = \frac{1 - |\lambda_f|^2 + \zeta}{1 + |\lambda_f|^2 + \zeta} = \frac{1 - |\bar{A}_f/A_f|^2}{1 + |\bar{A}_f/A_f|^2} = \frac{1 - |R_f|^2}{1 + |R_f|^2}, \quad (17)$$

$$A_{\text{CP}}^{\text{indir}} = -2 \left| \frac{q}{p} \right|^2 \frac{[(1 + |\lambda_f|^2) x \operatorname{Im}(\lambda_f) + (1 - |\lambda_f|^2) y \operatorname{Re}(\lambda_f)]}{\left(\left| \frac{q}{p} \right|^2 + |\lambda_f|^2 \right)^2}. \quad (18)$$

The above expression for $A_{\text{CP}}^{\text{indir}}$ reduces to the one commonly used [3] in the limit $R_f = 1$ in both the decay modes, since for CP-even final states like $\pi^+\pi^-$ and K^+K^- , we have $\lambda_f = -|(q/p)R_f|e^{i\phi}$ [6, 7]. Our expression is more general and needs to be used if the possibility of direct CP violation is to be taken into account. Even if the measured direct CP violation is very small, i.e. $|R_f| \approx 1.00$, it is possible that the phase of R_f is different for the two final states. This would make the value of λ_f different for the two final states. Indeed, as will be seen in Sec. 3, the measurements indicate $|R_\pi| \approx |R_K| \approx 1.00$ to within 1%, while $\operatorname{Arg}(R_\pi) - \operatorname{Arg}(R_K)$ can be large. Such a scenario would, of course, need the NP contribution to be of a very specific magnitude and phase. This important issue will be discussed later in Sec. 4 in detail.

Note that the effective lifetimes of the decay modes $D \rightarrow f$ and $\bar{D} \rightarrow f$ differ from τ_D by terms of $\mathcal{O}(x, y)$. However this does not change $A_{\text{CP}}^{\text{dir}}$, and the change in $A_{\text{CP}}^{\text{indir}}$ due to this is quadratic in x, y . Hence this difference can be neglected in our linear expansion. Integrating Eq. (16) over the observed normalized distribution of the proper decay time as measured in the $D \rightarrow f$ decay, we get

$$\langle A_{\text{CP}} \rangle = A_{\text{CP}}^{\text{dir}} + \frac{\langle t \rangle}{\tau_D} A_{\text{CP}}^{\text{indir}}. \quad (19)$$

Here $\langle t \rangle$ is average decay time that can be measured separately for each $D \rightarrow f$ decay mode. For the $\pi^+\pi^-$ and K^+K^- decay modes,

$$\langle A_{\text{CP}}(\pi) \rangle = A_{\text{CP}}^{\text{dir}}(\pi) + \frac{\langle t(\pi) \rangle}{\tau_D} A_{\text{CP}}^{\text{indir}}(\pi), \quad (20)$$

$$\langle A_{\text{CP}}(K) \rangle = A_{\text{CP}}^{\text{dir}}(K) + \frac{\langle t(K) \rangle}{\tau_D} A_{\text{CP}}^{\text{indir}}(K), \quad (21)$$

where $\langle t(\pi) \rangle, \langle t(K) \rangle$ are average decay times of D mesons for decays into the $\pi^+\pi^-$ and K^+K^- states, respectively. These average times are characteristics of specific experiments, the values for which have been shown in Table 1. Note that for the LHCb data, $\langle \bar{t} \rangle = (\langle t(K) \rangle + \langle t(\pi) \rangle)/2$ and $\Delta \langle t \rangle = \langle t(K) \rangle - \langle t(\pi) \rangle$.

¹ Note that this indeed is the definition of $A_{\text{CP}}^{\text{indir}}$ used in the experimental analysis of data [16, 19].

Quantity	Value	Reference
$\langle t(\pi) \rangle / \tau_D$	2.4 ± 0.03	CDF 2011*, CDF 2012 [15, 16]
$\langle t(K) \rangle / \tau_D$	2.65 ± 0.03	CDF 2011*, CDF 2012 [15, 16]
$\Delta \langle t \rangle / \tau_D$	$0.0983 \pm 0.0022 \pm 0.0019$	LHCb 2011 * (π -tagged) [17]
	$0.1119 \pm 0.0013 \pm 0.0017$	LHCb 2013 (π -tagged) [18]
	$0.018 \pm 0.002 \pm 0.007$	LHCb 2013 (μ -tagged) [19]
$\langle \bar{t} \rangle / \tau_D$	2.0826 ± 0.0077	LHCb 2011 * (π -tagged) [17]
	2.1048 ± 0.0077	LHCb 2013 (π -tagged) [18]
	$1.062 \pm 0.001 \pm 0.003$	LHCb 2013 (μ -tagged) [19]

Table 1: Experimental values of various quantities that appear in the determination of $A_{\text{CP}}^{\text{dir}}$ and $A_{\text{CP}}^{\text{indir}}$. We take $\tau_D = (0.41 \pm 0.0015)$ ps [5]. The data marked with a * are not used for the fit.

Subsequently, the difference CP asymmetry can be obtained as [51]

$$\begin{aligned}
\Delta A_{\text{CP}} &= \langle A_{\text{CP}}(K) \rangle - \langle A_{\text{CP}}(\pi) \rangle \\
&= A_{\text{CP}}^{\text{dir}}(K) - A_{\text{CP}}^{\text{dir}}(\pi) + \frac{\langle t(K) \rangle}{\tau_D} A_{\text{CP}}^{\text{indir}}(K) - \frac{\langle t(\pi) \rangle}{\tau_D} A_{\text{CP}}^{\text{indir}}(\pi).
\end{aligned} \tag{22}$$

As can be seen from Eqs. (17) and (18), $A_{\text{CP}}^{\text{indir}}$ depends on the final state f through its λ_f . Hence in general, indirect CP asymmetries in $D \rightarrow \pi^+ \pi^-$ and $D \rightarrow K^+ K^-$ can be different. Hence the equality $A_{\text{CP}}^{\text{indir}}(\pi) = A_{\text{CP}}^{\text{indir}}(K)$, as is generally used in the analyses of these channels, is only approximate. Note that this statement is independent of the mechanism of CP violation, since our analysis has been completely model-independent.

We would like to make a subtle point here. Equation (18) indeed agrees with the statement made in the literature [6] that in the absence of direct CP violation, the indirect CP violation is universal. However this statement needs to be interpreted with caution. It is true only if the absence of direct CP violation is taken to mean $R_f = 1$, both in magnitude as well as phase, for all modes (in any consistent phase convention). The absence of observable direct CP violation, however, only requires $|R_f| = 1$, which is not enough to guarantee this universality. On the other hand, the universality of indirect CP violation only needs R_f to be equal (in magnitude as well as phase) for all relevant decay modes, and its magnitude is immaterial.

The measured values of $\langle A_{\text{CP}}(\pi) \rangle$, $\langle A_{\text{CP}}(K) \rangle$, and ΔA_{CP} are shown in Table 2. The assumption of $A_{\text{CP}}^{\text{indir}}(\pi) = A_{\text{CP}}^{\text{indir}}(K)$ may also be responsible for the apparent discrepancy between the values of ΔA_{CP} measured at the LHCb through the pion-tagged and the muon-tagged samples. Note that the values of $\langle t(\pi) \rangle = \langle \bar{t} \rangle - \Delta \langle t \rangle / 2$ for the two samples are different, and so are the values of $\langle t(K) \rangle = \langle \bar{t} \rangle + \Delta \langle t \rangle / 2$. Indeed, we can write the difference $\delta(\Delta A_{\text{CP}}) \equiv (\Delta A_{\text{CP}})_\pi - (\Delta A_{\text{CP}})_\mu$ as

$$\begin{aligned}
\delta(\Delta A_{\text{CP}}) &= \left(\frac{\langle t(K) \rangle_\pi - \langle t(K) \rangle_\mu}{\tau_D} \right) A_{\text{CP}}^{\text{indir}}(K) - \left(\frac{\langle t(\pi) \rangle_\pi - \langle t(\pi) \rangle_\mu}{\tau_D} \right) A_{\text{CP}}^{\text{indir}}(\pi) \\
&= \left(\frac{(\Delta \langle t \rangle)_\pi - (\Delta \langle t \rangle)_\mu}{2\tau_D} \right) \left[A_{\text{CP}}^{\text{indir}}(K) + A_{\text{CP}}^{\text{indir}}(\pi) \right] + \\
&\quad \left(\frac{\langle \bar{t} \rangle_\pi - \langle \bar{t} \rangle_\mu}{\tau_D} \right) \left[A_{\text{CP}}^{\text{indir}}(K) - A_{\text{CP}}^{\text{indir}}(\pi) \right].
\end{aligned} \tag{23}$$

Quantity	Value (%)	Reference
$\langle A_{\text{CP}}(\pi) \rangle$	$-4.9 \pm 7.8 \pm 3.0$	E791 1997* [10]
	$4.8 \pm 3.9 \pm 2.5$	FOCUS 2000* [11]
	$1.9 \pm 3.2 \pm 0.8$	CLEO 2001* [12]
	0.04 ± 0.69	CDF 2011* [15]
	$-0.24 \pm 0.52 \pm 0.22$	Babar 2008 [13]
	$0.55 \pm 0.36 \pm 0.09$	Belle 2012 [14]
$\langle A_{\text{CP}}(K) \rangle$	$-1.0 \pm 4.9 \pm 1.2$	E791 1997* [10]
	$-0.1 \pm 2.2 \pm 1.5$	FOCUS 2000* [11]
	$0.0 \pm 2.2 \pm 0.8$	CLEO 2001* [12]
	$0.00 \pm 0.34 \pm 0.13$	Babar 2008 [13]
	-0.24 ± 0.41	CDF 2011* [15]
	$-0.32 \pm 0.21 \pm 0.09$	Belle 2012 [14]
ΔA_{CP}	$-0.82 \pm 0.21 \pm 0.11$	LHCb 2011* (π -tagged) [17]
	$-0.62 \pm 0.21 \pm 0.10$	CDF 2012 (π -tagged) [16]
	$-0.34 \pm 0.15 \pm 0.10$	LHCb 2013 (π -tagged) [18]
	$0.49 \pm 0.30 \pm 0.14$	LHCb 2013 (μ -tagged) [19]

Table 2: Experimental values of CP asymmetries measured at the experiments. The data marked with a * are not used for the fit.

The term on the last line would be missed if one assumes $A_{\text{CP}}^{\text{indir}}(\pi) = A_{\text{CP}}^{\text{indir}}(K)$. We shall revisit this quantitatively during our numerical analysis in the next section.

2.2 CP violating observables through effective lifetimes

The expansion of Eq. (10) to first order in x, y yields

$$\frac{d\Gamma}{dt}(D(t) \rightarrow f) \approx \frac{N_f}{2} e^{-\Gamma t} |A_f|^2 \left[1 + y\Gamma t \text{Re}(\lambda_f) - x\Gamma t \text{Im}(\lambda_f) \right]. \quad (24)$$

With $z_f \equiv x \text{Im}(\lambda_f) - y \text{Re}(\lambda_f)$, this could be written in the form [43]:

$$\frac{d\Gamma}{dt}(D(t) \rightarrow f) \propto e^{-\Gamma t} (1 - z_f \Gamma t). \quad (25)$$

The effective lifetime in the $D \rightarrow f$ mode is then

$$\tau_f \approx (1 - z_f)/\Gamma. \quad (26)$$

Since z_f depends on the decay mode $D \rightarrow f$ in general, the effective lifetimes measured in different modes can be different. These differences may be used to construct observables that are sensitive to CP violation in D decays.

For the decay $\bar{D} \rightarrow f$, Eq. (11) may also be written in another convenient form

$$\begin{aligned} \frac{d\Gamma}{dt}(\bar{D}(t) \rightarrow f) &= \frac{N_f}{2} e^{-\Gamma t} |\bar{A}_f|^2 \times \\ &\left[(1 + |\lambda_f^{-1}|^2) \cosh(y\Gamma t) + (1 - |\lambda_f^{-1}|^2) \cos(x\Gamma t) \right. \\ &\left. + 2 \text{Re}(\lambda_f^{-1}) \sinh(y\Gamma t) - 2 \text{Im}(\lambda_f^{-1}) \sin(x\Gamma t) \right]. \end{aligned} \quad (27)$$

After neglecting terms that are quadratic or higher powers in x, y , one gets

$$\frac{d\Gamma}{dt}(\bar{D}(t) \rightarrow f) \propto e^{-\Gamma t}(1 - \bar{z}_f \Gamma t), \quad (28)$$

with $\bar{z}_f \equiv x\text{Im}(\lambda_f^{-1}) - y\text{Re}(\lambda_f^{-1})$, so that the effective lifetime for this mode becomes

$$\bar{\tau}_f = (1 - \bar{z}_f)/\Gamma. \quad (29)$$

When the final state f is a CP eigenstate f_{CP} like $\pi^+\pi^-$ or K^+K^- , a CP-violating quantity can be constructed from the difference of the effective lifetimes of $D \rightarrow f_{\text{CP}}$ and $\bar{D} \rightarrow f_{\text{CP}}$ [43]:

$$\begin{aligned} A_\Gamma(f_{\text{CP}}) &\equiv \frac{\bar{\tau}_{f_{\text{CP}}} - \tau_{f_{\text{CP}}}}{\bar{\tau}_{f_{\text{CP}}} + \tau_{f_{\text{CP}}}} = \frac{1}{2}(z_{f_{\text{CP}}} - \bar{z}_{f_{\text{CP}}}) \\ &= \frac{1}{2}\left(x[\text{Im}(\lambda_{f_{\text{CP}}}) - \text{Im}(\lambda_{f_{\text{CP}}}^{-1})] - y[\text{Re}(\lambda_{f_{\text{CP}}}) - \text{Re}(\lambda_{f_{\text{CP}}}^{-1})]\right) \end{aligned} \quad (30)$$

clearly vanishes in the limit of CP conservation since $\lambda_{f_{\text{CP}}} = \pm 1$ in that case. This expression reduces to the one used in Ref. [3] in the limit $R_f = 1$, as expected. The relation $A_\Gamma = -A_{\text{CP}}^{\text{indir}}$ used in Ref. [3] is also valid only in this approximation, the actual relation being

$$A_\Gamma = -\frac{1}{4}A_{\text{CP}}^{\text{indir}} \left(|R_f| + \frac{1}{|R_f|} \right)^2. \quad (31)$$

Although the form of Eq. (30) seems different from the one given in Ref. [51], it is a result of expansions up to different orders in small quantities. In particular, we do not assume $\zeta(\equiv |q/p|^2 - 1)$ to be small, and keep terms to a higher power in it.

The quantities $A_\Gamma(\pi)$ and $A_\Gamma(K)$ have been measured separately [45, 46, 48] and as an average over the two modes [49, 50]; however, the errors are not small enough for a nonzero measurement. See Table 3.

For flavor-specific decays, where $D \rightarrow f$ is allowed but $\bar{D} \rightarrow f$ is not, λ_f vanishes and Eq. (10) gives

$$\frac{d\Gamma}{dt}(D(t) \rightarrow f) \approx N_f e^{-\Gamma t} |A_f|^2, \quad (32)$$

when terms with quadratic and higher powers of x, y are neglected. The average lifetime for such processes is clearly $\tau_{\text{FS}} \approx 1/\Gamma$. (Note that while taking $D \rightarrow \pi^- K^+$ to be a flavor-specific mode, the doubly Cabibbo-suppressed decay $D \rightarrow \pi^+ K^-$ has been neglected.) The quantity

$$\begin{aligned} y_{\text{CP}} &\equiv \frac{\tau_{FS}}{(\tau_{f_{\text{CP}}} + \bar{\tau}_{f_{\text{CP}}})/2} - 1 \approx \frac{1}{2}(z_{f_{\text{CP}}} + \bar{z}_{f_{\text{CP}}}) \\ &= \frac{1}{2}\left(x[\text{Im}(\lambda_{f_{\text{CP}}}) + \text{Im}(\lambda_{f_{\text{CP}}}^{-1})] - y[\text{Re}(\lambda_{f_{\text{CP}}}) + \text{Re}(\lambda_{f_{\text{CP}}}^{-1})]\right) \end{aligned} \quad (33)$$

is not necessarily CP-violating; however, it forms an important input for disentangling $z_{f_{\text{CP}}}$ and $\bar{z}_{f_{\text{CP}}}$ from the measurement of A_Γ . Note that the expression clearly reduces to the one used in Ref. [3] in the limit of $\bar{A}_f = A_f$ (or $R_f = 1$), and in the absence of CP violation, i.e. $\phi = 0$, one has $y_{\text{CP}} = y$.

Quantity	Value (%)	Reference
$A_\Gamma(\pi)$	$-0.28 \pm 0.52 \pm 0.15$	Belle 2007 [45]
	-0.049 ± 0.73	Babar 2008 [46]
$A_\Gamma(K)$	$0.15 \pm 0.35 \pm 0.15$	Belle 2007 [45]
	0.39 ± 0.46	Babar 2008 [46]
	$-0.59 \pm 0.59 \pm 0.21$	LHCb 2011 [48]
$A_\Gamma(\text{avg})$	$-0.03 \pm 0.20 \pm 0.08$	Belle 2012 [49]
	0.09 ± 0.27	Babar 2013 [50]
$y_{\text{CP}}(\pi)$	$0.5 \pm 4.3 \pm 1.8$	CLEO 2001 [12]
	$1.44 \pm 0.57 \pm 0.25$	Belle 2007 [45]
	$0.46 \pm 0.65 \pm 0.25$	Babar 2008 [46]
$y_{\text{CP}}(K)$	$3.42 \pm 1.39 \pm 0.74$	Focus 2000 [44]
	$-1.9 \pm 2.9 \pm 1.6$	CLEO 2001 [12]
	$1.25 \pm 0.39 \pm 0.25$	Belle 2007 [45]
	$1.60 \pm 0.46 \pm 0.17$	Babar 2008 [46]
	$1.12 \pm 0.26 \pm 0.22$	Babar 2009 [47]
	$0.55 \pm 0.63 \pm 0.41$	LHCb 2011 [48]
$y_{\text{CP}}(\text{avg})$	$1.11 \pm 0.22 \pm 0.11$	Belle 2012 [49]
	$0.72 \pm 0.18 \pm 0.12$	Babar 2013 [50]

Table 3: Measured values of A_Γ and y_{CP} .

Taking the CP-eigenstates to be $\pi^+\pi^-$ and the flavor-specific final state to be πK , The measurements of $y_{\text{CP}}(\pi)$ have so far been consistent with zero to within 2σ [12, 45, 46], while taking the CP-eigenstates to be K^+K^- , nonzero measurements of $y_{\text{CP}}(K)$ to more than 4σ level have been obtained [45, 46, 47]. Averaging over $\pi^+\pi^-$ and K^+K^- modes [49, 50] also gives a nonzero value at more than the 3σ level.

3 Numerical analysis

We now perform a χ^2 fit to the data on the $D - \bar{D}$ mixing and decay with an aim toward disentangling the contributions from CP-violation in mixing and in decay. The fit is performed to the four model-independent complex parameters $M_{12}, \Gamma_{12}, R_\pi$ and R_K . Here one has to be careful about the data to be included. We use the following prescription:

- For the data on $\langle A_{\text{CP}}(\pi) \rangle, \langle A_{\text{CP}}(K) \rangle$ and ΔA_{CP} , we use the experimental data as shown in Table 2 directly. The average decay times as given in Table 1 are used. Note that the data marked with a * are shown for the sake of completeness, but they are not used in the fit, either because they give too weak constraints, or because they have been used in later results by the same collaboration. This helps avoid double counting the same data.
- For the data on A_Γ and y_{CP} , we do not use the COMBOS fit [3] directly since it assumes equal values of these quantities in the $\pi^+\pi^-$ and K^+K^- channels. Whereas, as can be seen from Sec. 2.2, the difference between $A_\Gamma(\pi)$ and $A_\Gamma(K)$, as well as between $y_{\text{CP}}(\pi)$ and $y_{\text{CP}}(K)$, is of linear order in x, y when

Quantity	Value	Reference
x	$(0.80 \pm 0.29^{+0.09+0.10}_{-0.07-0.14})\%$	Belle 2007 [1]
y	$(0.33 \pm 0.24^{+0.08+0.06}_{-0.12-0.08})\%$	Belle 2007 [1]
$ q/p $	$0.86^{+0.30+0.06}_{-0.29-0.03}$	Belle 2007 [1]
φ	$(-14^{+16+5+2}_{-18-3-4})$ degrees	Belle 2007 [1]
x	$(0.16 \pm 0.23 \pm 0.12 \pm 0.08)\%$	Babar 2010 [2]
y	$(0.57 \pm 0.20 \pm 0.13 \pm 0.07)\%$	Babar 2010 [2]
$R_M = (x^2 + y^2)/2$	0.0130 ± 0.0269	[3]

Table 4: Experimental input for $D - \bar{D}$ mixing parameters.

$\lambda_\pi \neq \lambda_K$, and hence is relevant for our analysis here. We therefore use the data on $A_\Gamma(\pi), A_\Gamma(K), y_{\text{CP}}(\pi)$ and $y_{\text{CP}}(K)$ separately, as shown in Table 3. We have to contend with the problem that the most recent Belle and Babar results [49, 50] only report values of A_Γ and y_{CP} that are averaged over the $\pi^+\pi^-$ and K^+K^- modes. While using these data, we take the averaged values to be weighted averages, with weights proportional to the number of events in the two modes. However it would have been desirable to have the values of $A_\Gamma(\pi), A_\Gamma(K), y_{\text{CP}}(\pi)$ and $y_{\text{CP}}(K)$ separately, directly from the experiments, for a more accurate analysis.

- For the input from $D - \bar{D}$ mixing also, we do not use the HFAG [3] fit for $x, y, |q/p|, \varphi$ directly here since in addition to the $D - \bar{D}$ mixing data, it uses the data on A_{CP}, A_Γ and y_{CP} that have already been used above, which would have given rise to double counting of the data. We therefore only use the data from [1, 2], and the COMBOS average [3] for $R_M \equiv (x^2 + y^2)/2$ from the semileptonic D decays. To keep the number of fit parameters limited, we do not use the data on mixing parameters from the $K\pi$ or $K\pi\pi$ channel.

The χ^2 function is taken to be

$$\chi^2 = \sum_{i=1}^{32} \frac{(X_i - X_i^{\text{exp}})^2}{(\sigma_{X_i})^2}. \quad (34)$$

Here X_i, X_i^{exp} , and σ_{X_i} with $(i = 1, 2, 3, \dots, 32)$ represent the theoretical values, experimental values and corresponding experimental uncertainties, respectively, of the observables given in Tables 2, 3 and 4. We add the statistical and systematic errors in quadrature, and take all the measurements to be independent and uncorrelated. The MINUIT [52] subroutine is used for the minimization of χ^2 in the multidimensional parameter space. The best-fit values of the fit parameters are:

$$|M_{12}| = 0.0059 \text{ ps}^{-1}, \quad \text{Arg}(M_{12}) = 3.37, \quad |R_\pi| = 1.002, \quad \text{Arg}(R_\pi) = 3.82, \\ |\Gamma_{12}| = 0.0207 \text{ ps}^{-1}, \quad \text{Arg}(\Gamma_{12}) = 3.39, \quad |R_K| = 1.000, \quad \text{Arg}(R_K) = 3.20.$$

Note that the phases of M_{12} and Γ_{12} are convention dependent, however, the difference between them is independent of phase conventions. Since this difference is small, the magnitude of q/p at the best-fit point is still close to unity. It is also observed that the values of best fit for $|R_K|$ as well as $|R_\pi|$ do not deviate much from unity, so that the direct CP violation in both these decay modes is expected

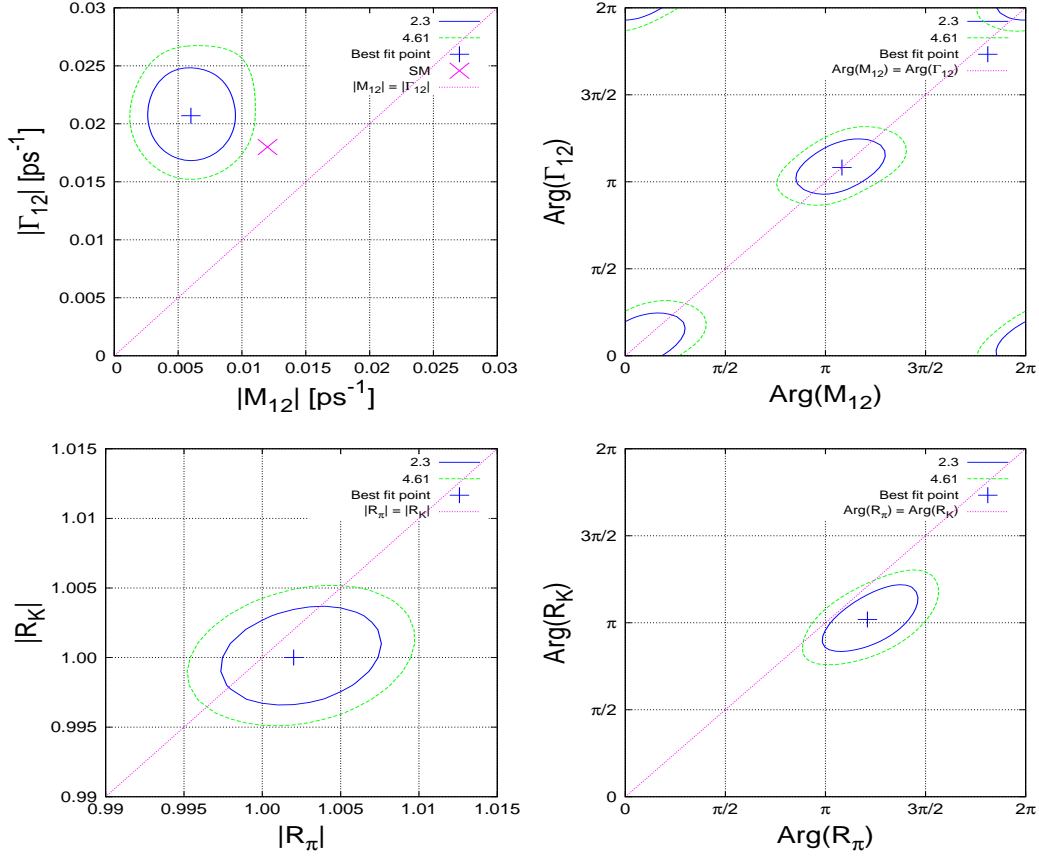


Figure 1: Parameter spaces favored by the current data on $D - \bar{D}$ mixing and decay. While displaying constraints on two parameters, the values of the other parameters are varied over all their allowed ranges to minimize χ^2 . The SM value of $(|M_{12}|, |\Gamma_{12}|)$ is taken from Ref. [30].

to be rather small. However, the phases of R_π and R_K at the best-fit point are significantly different. This will be relevant in our discussion later.

The fit is rather good: at the best-fit point, $\chi^2/\text{dof} = 27.6/24$. It is interesting that even if we impose a further restriction of $|\lambda_\pi| = |\lambda_K|$, which would correspond to $|R_\pi| = |R_K|$, the fit still stays almost as good, with $\chi^2/\text{dof} = 28.1/25$. The values of the best-fit parameters are also very similar. This indicates that around the best-fit point, the CP-violation through mixing alone as well as CP-violation through decay alone is very small, so the CP violation observed is mainly through the interference between mixing and decay. Further insisting on identical values for the magnitudes as well as phases of λ_π and λ_K , however, worsens the fit to $\chi^2/\text{dof} = 37.3/26$. This indicates the tension of the data with the scenario of equal CP violation in the two decays. As indicated in the previous section, this implies significantly different values of $A_{\text{CP}}^{\text{indir}}$ in the two decays.

To make quantitative statements about the significance of our observations above, we show the parameter spaces favored by the data in Fig. 1. The contours shown in the figure correspond to $\Delta\chi^2 = 2.3$ (68% C.L.) and $\Delta\chi^2 = 4.61$ (90% C.L.). The following observations may be made from the figure:

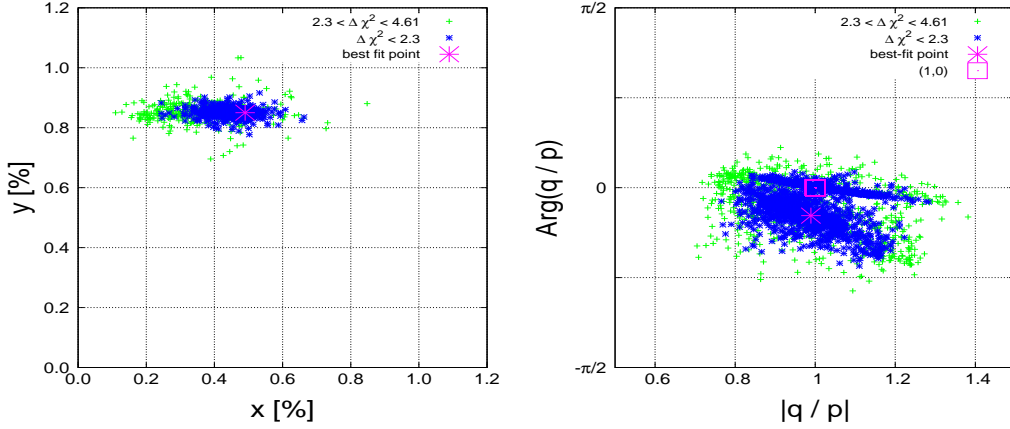


Figure 2: Mixing parameters in $D - \bar{D}$ mixing, applicable to both, $\pi^+\pi^-$ and K^+K^- decays.

- $|\Gamma_{12}| > |M_{12}|$ in the whole of the region allowed to 90% C.L.. Indeed, the data seem to favor the region with $|\Gamma_{12}|$ equal to a few times $|M_{12}|$.
- The phase between M_{12} and Γ_{12} is compatible with zero, although deviations of up to ≈ 0.9 radians are possible. The SM prediction for this phase would be $\mathcal{O}(0.1\%)$, so much larger values for this phase are still allowed by the data.
- The 90% allowed values for both $|R_\pi|$ and $|R_K|$, for both $\pi^+\pi^-$ and K^+K^- modes are consistent with unity. While a deviation of $\approx 1\%$ from unity is allowed for $|R_\pi|$, the value of $|R_K|$ is restricted to be within $\approx 0.5\%$ of unity. This indicates that the direct CP violation $A_{\text{CP}}^{\text{dir}}$ is restricted to a fraction of a per cent.
- The relative phase between R_π and R_K is a convention-independent, physical quantity. Data seem to prefer different phases for R_π and R_K . This indicates that although $|R_\pi| \approx |R_K|$ is allowed, $R_\pi = R_K$ is disfavored. This would further imply $\lambda_\pi \neq \lambda_K$, and consequently $A_{\text{CP}}^{\text{indir}}(\pi) \neq A_{\text{CP}}^{\text{indir}}(K)$.

The derived quantities $x, y, |q/p|$ and $\varphi \equiv \text{Arg}(q/p)$ that are commonly used to describe the $D - \bar{D}$ mixing are shown in Fig. 2. The figure indicates the following:

- The values of both x and y are positive to 90% C.L..
- Although $|q/p|$ is consistent with unity, a variation in the range (0.7, 1.3) is still allowed to 90% C.L.. This implies that significant CP violation through mixing is allowed. Note that our fit gives higher values of $|q/p|$ as compared to the one in Ref. [3]; however, there are differences in the two fit procedures. We have used only a subset of the data used therein, but have taken care of possibly different values of A_Γ and y_{CP} in the $\pi^+\pi^-$ and K^+K^- modes.
- The phase φ is restricted to be in the range $(-0.9, 0.3)$ to 90% C.L.. This quantity is of course phase-convention dependent, and is physically meaningful only when compared with the phases of R_π or R_K . So we shall not discuss it further.

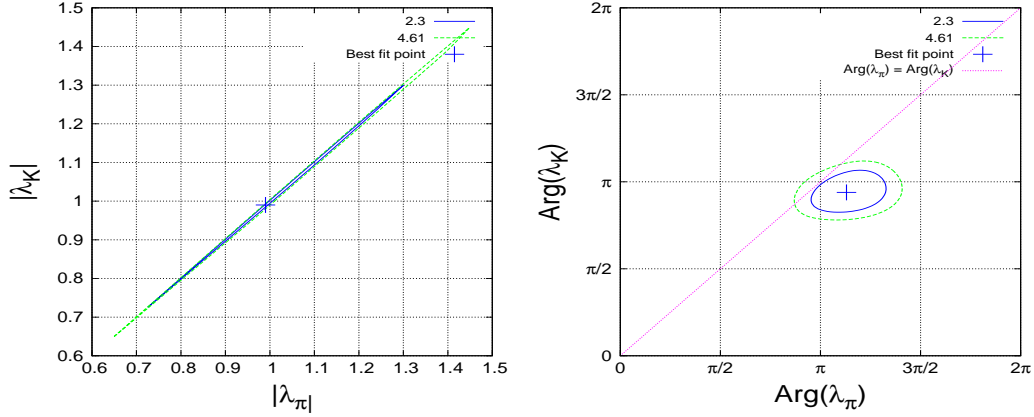


Figure 3: Constraints on the magnitudes and phases of λ_π and λ_K .

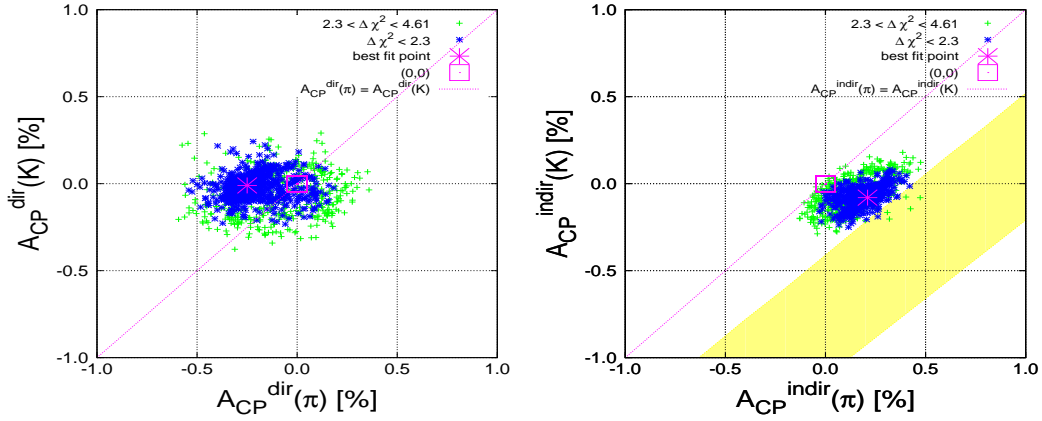


Figure 4: Constraints on direct and indirect asymmetries in $D \rightarrow \pi\pi$ and $D \rightarrow KK$ from the data in Tables 2, 3 and 4. The yellow (gray) band in the plot on the right corresponds to the values that will reconcile the ΔA_{CP} measurements through the pion-tagged and muon-tagged samples at the LHCb to within 1σ .

Let us now explore the extent of CP violation through the interference of mixing and decay. This may be parametrized through the imaginary part of $\lambda_\pi \equiv (q/p)R_\pi$ and $\lambda_K \equiv (q/p)R_K$. While the phases of q/p , R_π and R_K shown above are convention dependent, the phases of λ_π and λ_K are physical quantities, independent of the phase convention used. We show the allowed ranges of the magnitudes and phases of λ_π and λ_K in Fig. 3.

- The magnitudes of λ_π and λ_K are highly correlated: $|\lambda_\pi| \approx |\lambda_K|$. This is expected, since $|\lambda_f| = |(q/p)R_f|$, wherein the quantity $|q/p|$ is common to both the decay modes, and $|R_\pi|$ and $|R_K|$ are both very close to unity.
- On the other hand, different phases for λ_π and λ_K seem to be preferred. This leads to a difference in the values of λ_π and λ_K , and hence different values of A_{CP}^{indir} in the two modes.

With the allowed ranges of parameters as determined above, we present the allowed values of A_{CP}^{dir} and A_{CP}^{indir} in the form of a scatter plot in Fig. 4. We can observe the following:

- At the best-fit point, we have

$$\begin{aligned} A_{\text{CP}}^{\text{dir}}(\pi) &= -0.0024, & A_{\text{CP}}^{\text{indir}}(\pi) &= 0.0021, \\ A_{\text{CP}}^{\text{dir}}(K) &= -0.0001, & A_{\text{CP}}^{\text{indir}}(K) &= -0.0008, \end{aligned}$$

so that the data favors different indirect CP asymmetries in these two modes.

- While the direct CP violation $A_{\text{CP}}^{\text{dir}}$ in the $\pi^+\pi^-$ mode is restricted to be less than a per cent, that in the K^+K^- mode is restricted even more severely, to be less than half a per cent. These allowed values are still much larger as compared to the SM expectations.
- Indirect CP violation to the extent of half a per cent is still allowed for $\pi^+\pi^-$, while in the case of K^+K^- it can be maximum up to a quarter of a per cent. More importantly, different values of $A_{\text{CP}}^{\text{indir}}$ in these modes are highly preferred by the data.
- The shaded band shows that region in which the apparent discrepancy between the ΔA_{CP} measurements from the pion-tagged and muon-tagged sample is resolved to within 1σ . As expected, the resolution favors significantly different values of $A_{\text{CP}}^{\text{indir}}$, which is consistent with the results of our fit, almost to $\sim 1\sigma$. Referring back to Eq. (23), the large coefficient of the $[A_{\text{CP}}^{\text{indir}}(K) - A_{\text{CP}}^{\text{indir}}(\pi)]$ (see Table 1) allows such an explanation of the apparent discrepancy through a moderate difference in the indirect CP asymmetries in the two decay modes.

Before continuing, we also present the information on the quantities

$$\begin{aligned} z_\pi &\equiv x \operatorname{Im}(\lambda_\pi) - y \operatorname{Re}(\lambda_\pi), & \bar{z}_\pi &\equiv x \operatorname{Im}(\lambda_\pi^{-1}) - y \operatorname{Re}(\lambda_\pi^{-1}), \\ z_K &\equiv x \operatorname{Im}(\lambda_K) - y \operatorname{Re}(\lambda_K), & \bar{z}_K &\equiv x \operatorname{Im}(\lambda_K^{-1}) - y \operatorname{Re}(\lambda_K^{-1}), \end{aligned} \quad (35)$$

obtained from the measurements of the quantities A_Γ and y_{CP} . It may be observed from Fig. 5 that the favored regions in the $(z_\pi - \bar{z}_\pi)$ and $(z_K - \bar{z}_K)$ parameter space are quite different; they have only a small overlap. This is in consonance with our overall observation that the data indicate unequal amount of CP violation in $\pi^+\pi^-$ and K^+K^- modes. It is therefore important that the measurements of $A_\Gamma(\pi), A_\Gamma(K), y_{\text{CP}}(\pi)$ and $y_{\text{CP}}(K)$ be available separately, without averaging over the $\pi^+\pi^-$ and K^+K^- modes, and analyzed without the assumption of their equality.

4 Feasibility and implications of nonuniversal $A_{\text{CP}}^{\text{indir}}$

Our analysis as such does not depend on whether we have only SM, or whether NP is present in addition. However within the SM, even given the uncertainties due to long-distance contributions, it is very difficult to get CP violation of the order of 1% or larger. Indeed, if $|q/p|$ actually differs substantially from unity, it will need a large phase difference between M_{12} and Γ_{12} , which does not seem possible in the SM. Also, getting significantly different phases for the quantities R_π and R_K , as indicated by the data, is not something the SM can do. We therefore interpret our results in terms of the demands they make on NP models, and the observations required for identifying such NP. To compare our results with previous analyses, here we present our arguments in terms of the language and notation used in Refs. [5, 6, 7].

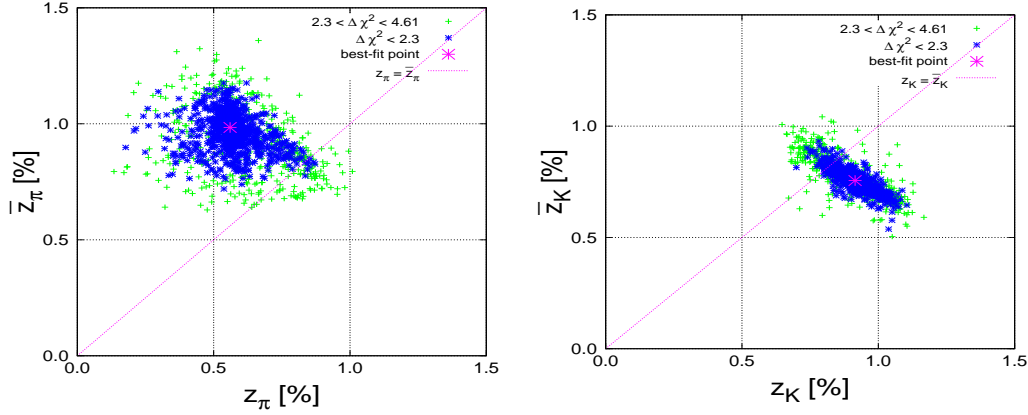


Figure 5: Bounds on the parameters z_π , \bar{z}_π , z_K , and \bar{z}_K , from the measurements of $A_\Gamma(\pi)$, $A_\Gamma(K)$, $y_{\text{CP}}(\pi)$ and $y_{\text{CP}}(K)$.

In the notation of the Particle Data Group [5], the decay amplitudes A_f and \bar{A}_f [see Eq. (8)] are written as

$$\begin{aligned} A_f &= A_f^T e^{+i\phi_f^T} \left[1 + r_f e^{i(\delta_f + \phi_f)} \right], \\ \bar{A}_f &= A_f^T e^{-i\phi_f^T} \left[1 + r_f e^{i(\delta_f - \phi_f)} \right], \end{aligned} \quad (36)$$

where A_f^T is the leading tree amplitude with its corresponding phase ϕ_f^T , while r_f is the ratio of the subleading to the leading amplitude, with the corresponding strong and weak phase differences δ_f and ϕ_f , respectively. This notation may be matched to ours by using

$$R_f = \frac{\bar{A}_f}{A_f} = e^{-2i\phi_f^T} \left[\frac{1 + r_f e^{i(\delta_f - \phi_f)}}{1 + r_f e^{i(\delta_f + \phi_f)}} \right]. \quad (37)$$

The approximation $r_f \ll 1$ used in [5, 6] leads to universal indirect CP violation ($= a^m + a^i$) that is independent of r_f , δ_f and ϕ_f . As a result, the quantities A_Γ and y_{CP} are also universal, i.e. identical for the $\pi^+\pi^-$ and K^+K^- modes.

Although the approximation $r_f \ll 1$ is valid in the SM, it is not guaranteed to be true in the presence of NP. Higher-order terms in r_f give nonuniversal contributions to $A_{\text{CP}}^{\text{indir}}$ through different values for λ_f [7]. The value of r_f is bounded by the CP violation measurements themselves: since $A_{\text{CP}}^{\text{dir}}(f) \sim \mathcal{O}(0.01)$, we have $|R_f| \sim 1 + \mathcal{O}(0.01)$, and hence $r_f \sin \delta_f \sin \phi_f \sim \mathcal{O}(0.01)$, for both the decay modes. On the other hand, in order to have a significant nonuniversality in $A_{\text{CP}}^{\text{indir}}$ through $\lambda_\pi \neq \lambda_K$, we need $\text{Arg}(\lambda_\pi) \neq \text{Arg}(\lambda_K)$, since our fit suggests equal magnitudes for these two quantities (see Fig. 3). Using the results in [7], the relevant nonuniversality may be expressed in terms of

$$\text{Arg}(\lambda_K) - \text{Arg}(\lambda_\pi) \approx 2r_\pi \cos \delta_\pi \sin \phi_\pi - 2r_K \cos \delta_K \sin \phi_K, \quad (38)$$

so that at least one of the two quantities $r_f \cos \delta_f \sin \phi_f$ should be significantly large, i.e. $\mathcal{O}(0.1 - 1)$. For both the above constraints to be satisfied simultaneously, we need $\sin \phi_f \sim \mathcal{O}(1)$, $r_f \sin \delta_f \sim \mathcal{O}(0.01)$, and $r_f \cos \delta_f \sim \mathcal{O}(0.1 - 1)$, for at least one of the final states $\pi^+\pi^-$ and K^+K^- .

The above argument suggests that for the condition implied by our best-fit point to be met, one needs an enhanced subleading amplitude with a large relative weak phase and a small relative small phase compared to the leading term, for at least one of the two final states. While the first two conditions may be generically satisfied by a NP model that is not too constrained from other measurements, the smallness of the relative strong phase may seem to be rather fine-tuned, since the NP operators typically differ from the leading ones in their color and chirality structure [7]. Explicit calculations or direct measurements of these strong phase differences are not available; recent analysis of the $\pi\pi$ scattering data in the context of SM [53] indicates large strong phases, but note that what is needed here is a small *difference* in the strong phases of the leading and subleading contributions. On the other hand, the magnitude and phase of the NP contribution should conspire such that $|R_f| \approx 1.00$ to within 1%. The NP scenario required here is thus rather fine-tuned, but is not ruled out, and hence should not be ignored. Further, note that if the difference between the LHCb measurements with the pion-tagged and muon-tagged samples is real, this is perhaps the only mechanism that can account for it. In fact, the measurement of nonuniversality of $A_{\text{CP}}^{\text{indir}}$ itself would give direct information on the smallness or largeness of this phase that appears in all the CP-violating observables.

The NP scenario described above, which gives large nonuniversality in $A_{\text{CP}}^{\text{indir}}$, is mandated if the best-fit point obtained from our fit indeed turns out to be the right one with future data. However even if the future data were to indicate smaller nonuniversality in $A_{\text{CP}}^{\text{indir}}$ than the best-fit point obtained our fit, our analysis in Sec. 2 still stays relevant. It gives generalized expressions for the CP asymmetry A_{CP} and the related quantities A_Γ, y_{CP} [see Eqs. (17), (18), (30), (33)] that are valid even with nonuniversal $A_{\text{CP}}^{\text{indir}}$. The expressions prevalent in the standard literature [3, 5] do not take this possibility into account.

The nonuniversality of A_Γ and y_{CP} has been explicitly calculated in [51], albeit with different expansion parameters than the ones considered here. We believe that our expansion parameters (x and y) are more well motivated since they have been measured to be small. Moreover, while obtaining the final expression for ΔA_{CP} , Ref. [51] used the assumption of universality of the phase of λ_f , thus restricting its domain of validity.

5 Summary and conclusion

The recent measurements of the CP asymmetries in $D/\bar{D} \rightarrow \pi^+\pi^-, K^+K^-$ modes, and the difference in the CP asymmetries in these modes (the so-called difference CP asymmetry) have yielded values differing from the SM expectations. Moreover, the difference CP asymmetries measured at the LHCb through the pion-tagged and the muon-tagged samples differ substantially. We examine these data in a model-independent manner to discern the nature of CP violation involved therein and to find a resolution for the above discrepancy.

By performing a fit to the data on $D - \bar{D}$ mixing, CP asymmetries in $D/\bar{D} \rightarrow \pi^+\pi^-, K^+K^-$ modes, as well as the related quantities A_Γ and y_{CP} in these channels, we find the following: (i) The CP violation through decay-only in both the modes is restricted to be less than $\mathcal{O}(0.5\%)$, although this limit still allows values much larger than those permitted by the SM. (ii) The CP violation through mixing-only, on the other hand, can be quite large — the value of $|q/p|$ can differ from unity by

$\sim \mathcal{O}(10\%)$. (iii) The CP violation through the mixing-decay interference may play an important role in the $D/\bar{D} \rightarrow \pi^+\pi^-, K^+K^-$ modes. The phases of the quantities λ_π and λ_K tend to differ substantially.

In the language of direct and indirect CP asymmetries, as used while presenting the recent experimental data, the direct CP asymmetries in both, $\pi^+\pi^-$ and K^+K^- modes is restricted to be less than $\mathcal{O}(0.5\%)$, and these two asymmetries can have different values. The indirect CP asymmetries in these two modes also can differ substantially in certain scenarios. Indeed we demonstrate that, mathematically speaking, different direct CP asymmetries imply different indirect CP asymmetries, unless there are accidental cancellations. We therefore emphasize that taking the indirect CP asymmetries in these two channels to be equal is an approximation, one that may be valid, but that should be checked with data.

It turns out that the possibility of nonuniversal indirect CP asymmetry also allows a partial reconciliation between the seemingly different difference CP asymmetries measured through the pion-tagged and muon-tagged samples at the LHCb. The analyses for these decay modes in terms of the direct and indirect CP asymmetries should therefore be performed without the usual assumption of equal indirect CP asymmetries in the two modes.

Our formalism also allows us to express the quantities A_Γ and y_{CP} in a symmetric form, $A_\Gamma = (z - \bar{z})/2$ and $y_{CP} = (z + \bar{z})/2$, without having to assume equal magnitudes as well as phases for $A(\bar{D} \rightarrow f)/A(D \rightarrow f)$ for the $\pi^+\pi^-$ and K^+K^- channels. This also indicates that the data on these quantities in the $\pi^+\pi^-$ and K^+K^- channels should be presented separately, since these quantities can be different in these two modes and an averaging might lose information critical for ascertaining the presence and nature of any NP present.

A significant nonuniversality in A_{CP}^{indir} would require the subdominant amplitude in $D \rightarrow f$ decay to be comparable in magnitude to the dominant one, as well as a small strong relative phase and a large weak relative phase between these two amplitudes. This scenario appears fine-tuned, given the theoretical expectation of large relative strong phases; however, there is no direct calculation or measurement of the strong phase, so it is not ruled out. Therefore, it is important to take into account the possibility of such a NP that could give rise to significant nonuniversality in A_{CP}^{indir} , A_Γ , and y_{CP} . The generalized analysis and expressions presented in this paper then need to be used instead of the ones in standard literature that assume universality of A_{CP}^{indir} . Indeed, such a generalized analysis will be useful in measuring the extent of the nonuniversality itself, and testing of the theoretical expectation of a large strong relative phase.

Since using the generalized analysis (by removing just one assumption) offers the possibility of explaining the current data as well as probing NP signals with future data, this opportunity should not be missed. This implies that the averaging of A_Γ, y_{CP} values in $\pi^+\pi^-, K^+K^-$ modes at the B factories, as well as the averaging of ΔA_{CP} in pion-tagged and muon-tagged modes in LHCb, should be avoided. With large amounts of data around the corner from the LHC upgrade and the super-B factory, statistics will cease to be the limiting factor, and one can probe possible NP in the CP violation in D decays in a clean way.

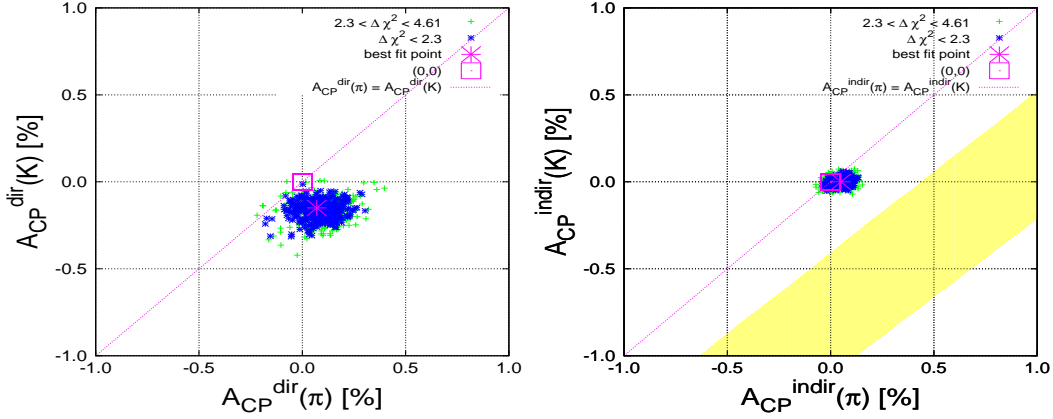


Figure 6: Constraints on direct and indirect asymmetries in $D \rightarrow \pi\pi$ and $D \rightarrow KK$ from the current data, with the new LHCb 2013 [54] results added. The yellow (gray) band in the plot on the right corresponds to the values that will reconcile the ΔA_{CP} measurements through the pion-tagged and muon-tagged samples at the LHCb to within 1σ .

Note Added

While this article was under review, the LHCb Collaboration announced undated results on A_Γ [54]: $A_\Gamma(K) = (-0.035 \pm 0.062 \pm 0.012)\%$, $A_\Gamma(\pi) = (0.033 \pm 0.106 \pm 0.014)\%$. Adding these measurements to our fit, we find that (see Fig. 6) the ability of nonuniversal- A_{CP}^{indir} to help reconcile the ΔA_{CP} measurements through the pion-tagged and muon-tagged samples at the LHCb is rather restricted by the new data. However the last word on the CP violation in D meson system is yet to be written [55], and a complete analysis should take into account the possibility of different indirect CP asymmetries in the $\pi^+\pi^-$ and K^+K^- channels.

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